Exponential Functions - An Algebra Review (Section 1.5) **Date** ______________

After the linear function, the exponential function is the most important function for the application of mathematics to real world problems.

Let's begin with an example.

Example. Two situations are described below. For each situation, complete the table of values of the population P of the town at the indicated number of years t, and then find a function P(t) for the population of the town in terms of the years t.

a. The town has a population of 1000 in year 0, and grows at a rate of 50 people per year.

Years	0	1	2	3	4	Formula
Population						P(t) =

b. The town has a population of 1000 in year 0, and grows at a rate of 5% per year.

Years	0	1	2	3	4	Formula
Population						P(t) =

The function we have discovered in part a is a ______________________ function.

The function we have discovered in part b is an ______________________ function. The function tells us that the initial value of the function (when t = 0) was ______________, the rate of growth was __________ (or __________ %) and the growth factor was ______________.

Graph the two functions on your calculator to see how they compare.

Example: Look at the values in the following table

x	0	1	2	3	4
f(x)	1875	1125	675	405	243

Do the values in the table represent a linear function? Why or why not?

Do the values in the table represent an exponential function? Why or why not?

Find a function for the values in the table. $f(x) =$ ____________________. Sketch a graph of f on your calculator. Why does this graph look different than the exponential function in the earlier example?

This is an example of <u>exponential decay</u>, not <u>exponential growth</u>. We can tell because the base of the exponential function is ____________________. The rate of decay is __________ or __________%, and the decay factor is ____________.

The General Exponential Function: We say that P is an exponential function of t with base a if:

$$P = P_0 a^t,$$

where P_0 is the ______________________________ (when $t = 0$), and a is the ______________________________, that is, the factor that P changes when t increases by 1. We have **exponential growth** when __________, and **exponential decay** when __________.

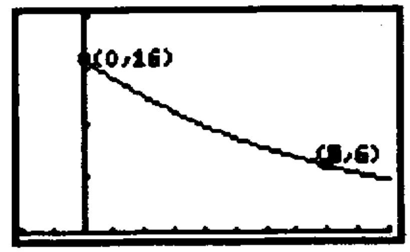
The factor a is given by:

$$a = 1 + r$$

where r is the decimal representation of the percent rate of change. If $r > 0$, then we have ________________________ and if $r < 0$, we have ________________________

Examples:

1. Find an exponential function in the form $f(x) = a \cdot b^x$ for the graph shownto the right. (Round the value of b to 3 decimal places.)



$f(x) =$ ____________________

This function has a growth/decay rate of ____________

2. The company that produces Cliff Notes (the abridged versions of classic literature) was started in 1958 with $4000 and sold in 1998 for $14,000,000. Find the annual percent increase in the value of this company over the 40 years, and write a function for the value of the Cliff Notes company in terms of the years since 1958.

3. Without your calculator, sketch the graph of the following "series of functions". Label the y-intercept and the horizontal asymptote for each. Check your answers with your calculator.

a. $y = (0.6)^x$ b. $y = 8(0.6)^x$ c. $y = -8(0.6)^x$ d. $y = -8(0.6)^x + 15$

You may remember (hopefully), from your previous algebra course, that there is a very important base when studying exponential functions. In fact, this is the most commonly used base when using exponential functions.

This base is the number ________, which is approximately equal to ____________.

Since this number is between 2 and 3, the graph of $y = e^x$ should be between $y = 2^x$ and $y = 3^x$. Try it!

The base e is used so often that it is called the <u>natural base</u>, and the function $y = e^x$ is called the <u>natural exponential function</u>. (We will get an indication of "why" when we apply calculus to the exponential function!)

The Natural Logarithmic Function - An Algebra Review (Section 1.6) **Date:** ____________

We know how to algebraically solve equations like $400 = 5 \cdot x^7$.

$x \approx$ ________

But how about an equation like $400 = 5 \cdot 7^x$? In this equation the variable x is an exponent, and therefore, it is called an **exponential equation.** Solving such an equation requires an understanding of logarithms.

We'll come back to this problem.

Definition of the Natural Logarithm.

The **natural logarithm of x,** written **ln x** is defined as: **ln x = c means** ____________.

In other words, **ln x** is the ____________________________.

Note: The only logarithm we will be using in this course is the natural logarithm.

Using this definition, evaluate the following (without a calculator).

1. $\ln 1 =$ ________ because ____________________
2. $\ln e =$ ________ because ____________________
3. $\ln\left(\frac{1}{e}\right) =$ ________ because ____________________
4. $\ln\sqrt{e} =$ ________ because ____________________
5. $\ln(-10) =$ ______________ because ______________________________
6. $\ln 0 =$ ______________ because ______________________________

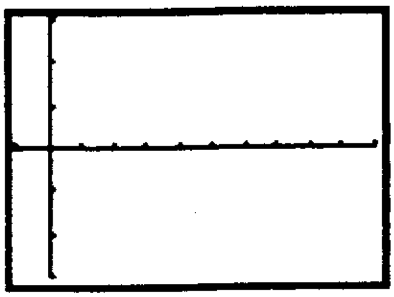
You may also remember (hopefully), some properties of logarithms.

1. $\ln(A \cdot B) =$ ____________________
2. $\ln\left(\frac{A}{B}\right) =$ ____________________
3. $\ln(A^n) =$ ____________________
4. $\ln e^x =$ ____________________
5. $e^{\ln x} =$ ____________________

Example: Apply the properties of logarithms to the expression $\ln\left(\frac{3\sqrt{x}}{yz^4}\right)$ in terms of ln x, ln y, ln z.

$$\ln\left(\frac{3\sqrt{x}}{yz^4}\right) =$$

Using your calculator, sketch a graph of $f(x) = \ln x$, and answer the following questions concerning the function.



1. State the Domain: ____________ Range: ____________

2. What's the x-intercept? ____________

3. When is $f(x) > 0$? __________ $f(x) < 0$? ______________

4. When is $f'(x) > 0$? __________ $f'(x) < 0$? ______________

5. When is $f''(x) > 0$? __________ $f''(x) < 0$? ______________

Now, let's go back to the example of solving the exponential equation $400 = 5 \cdot 7^x$.

Examples:

1. Last class, the function that we developed to solve the Cliff Notes problem was $P(t) = 4000(1.226)^t$, where t represented the number of years after 1958. In what year did the value of the Cliff Notes company reach 5 million dollars?

2. Solve the equation: $24 = 5e^{3t}$.

Exponential Functions with Base e

Last class period we saw that an exponential function with base "a" has a formula

$$P = P_0 a^t.$$

Recall that P_0 represents ____________________, and a represents the ____________________.

If $a > 1$, then we have ____

If $0 < a < 1$, then we have ____

This model is generally used when the growth rate is measured "per unit of time"; for example, a population is increasing 5% per year.

If, however, a quantity is increasing (or decreasing) at a **continuous growth rate,** the function is usually written in the form

$$P = P_0 e^{kt}.$$

What we have done, is replaced the "a" with an "e^k". Let's see what effect this has.

1. Let $a = e^k$, Solve for k. $k =$ ____________

2. If $a > 1$, then k __________, and if $0 < a < 1$, then k ______________.
3. The value of k is called the **continuous growth rate.**

Examples:

1. Convert the function $P = 700(1.07)^t$ to the form $P = P_0 e^{kt}$. Interpret the results.

2. Convert the function $P = 2000e^{-0.04t}$ to the form $P = P_0 a^t$. Interpret the results.